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Loop-Gap Resonator: A Lumped Mode Microwave Resonant Structure

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Abstract —**Loop-gap resonators are newly developed microwave resonant structures with a field configuration that is intermediate between lumped and distributed. Typical characteristic dimensions are of the order of 1/10 of the resonant wavelength, and typical Q's are of the order of 1600–2000 in the frequency range of 1–4 GHz. Data are presented for Q's and frequencies for a series of resonators of various dimensions and compared with theory. Various coupling and frequency tuning techniques are discussed, and results of experiments are reported. Results of preliminary application of the structure in microwave filters and oscillators are presented. Loop-gap resonators provide a useful design alternative, it is concluded, to dielectric and surface acoustic-wave resonators at low microwave frequencies.**

I. Introduction and General Description of the Resonator

At low frequency microwave bands of L and S, the choice of resonators presents a problem because of the large size of resonant cavities and the high loss of lumped-element circuits. Miniature lumped-element resonant circuits were described in [1], which are composed of an interdigital capacitor and a loop inductor etched on an MIC substrate. They were designed for X-band frequencies with a Q of about 700. Here we describe a resonant structure with a lumped-mode field configuration where the transmission line between the inductive and capacitive elements is eliminated; therefore, these elements are in juxtaposition which results in low loss.

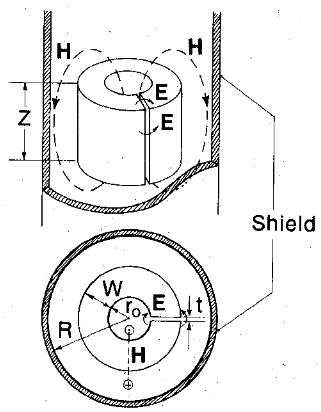

Fig. 1. The loop-gap resonator and cross-sectional view.

The loop-gap resonator is shown in Fig. 1. It consists of a conductive cylindrical loop cut by one or more longitudinal slots (or gaps). In this paper, only single-gap resonators are discussed. The structure is shielded by a conductive cylinder coaxial with the resonator. The resonators discussed here are machined from brass stock and silver plated for better Q. The resonator is supported in position within the cylindrical shield by the use of a semi-annular piece of Rexolite and plastic screws.

The electric fields, as shown in Fig. 1, are supported by the gap with the magnetic fields surrounding the loop. The conduction current, which flows in a circumferential direction on surfaces of the loop, transforms into displacement current in the gap.

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TABLE I
PHYSICAL DIMENSIONS (IN MILLIMETERS) AND MEASURED AND CALCULATED RESONANT FREQUENCIES IN (MHz) AND QUALITY FACTORS FOR FOUR SILVER-PLATED BRASS RESONATORS

RES. NO.	r_0	W	t	R	Z	f_0 MEAS.	f_0 CALC	Q_0 MEAS.	Q_0 CALC.
1	6.3	2.5	0.228	15.0	17.8	1413.0	1352.3	2050	2320
2	3.2	2.4	0.330	10.2	10.2	3050.8	3055.3	1800	1870
3	6.3	6.3	0.254	19.0	30.5	998.5	927.1	1600	1930
4	1.5	0.8	0.330	4.0	5.1	8877.0	9072.0	1100	1410

structure in magnetic resonance spectroscopy (NMR and ESR) [2]–[4]. In [2], the designation "split-ring resonator" was used. A structure based on the same resonance principle was used for acceleration of charged particles [5]–[7], utilizing the high electric-field intensity of the gap. A structure has been in use in magnetron amplifiers [8] that uses a similar resonance principle for acceleration of electrons. It was named "hole and slot resonator" because of its particular geometry.

The purpose of this paper is to present general characteristics and some experimental applications of loop-gap resonators. Theoretical analysis of the resonator has been completed [9] and will eventually be published. Some of the results of this analysis are used here as a design aid and also to check the experimental results.

Table I shows physical dimensions in millimeters, and measured and calculated resonant frequencies and Q factors for four silver-plated brass resonators used in the measurements presented here (for calculations, see Section IV).

II. COUPLING

A loop-gap resonator can be coupled to external circuits by both inductive and capacitive means. An inductive loop can be used for coupling [2], [3], utilizing the magnetic fields at either end of the resonator. Fig. 2 shows this coupling method with its field configuration and equivalent circuit. Fig. 2(c) shows the coupling coefficient versus loop distance for resonator No. 2 in Table I, using a coupling loop with a radius of 3 mm connected to a 50-Ω coaxial line.

The loop-gap resonator can be magnetically coupled to a microstrip transmission line as shown in Fig. 3. The figure shows the field configuration for this coupling method and its equivalent circuit. This coupling arrangement makes the loop-gap resonator compatible with microwave integrated circuits. For resonator No. 2 in Table I, and a 50-Ω microstrip line on a Teflon substrate with a thickness of 0.8 mm, a coupling coefficient of unity was obtained when the closest distance of the resonator from the line was 0.5 mm.

Capacitive coupling to a coaxial line is possible by means of a monopole antenna probe in the proximity of the gap as shown in Fig. 4. In this method, coupling occurs through the interaction of the gap's fringe electric fields with the monopole. The probe is an extension of the central wire in the coaxial cable as shown in Fig. 4. A

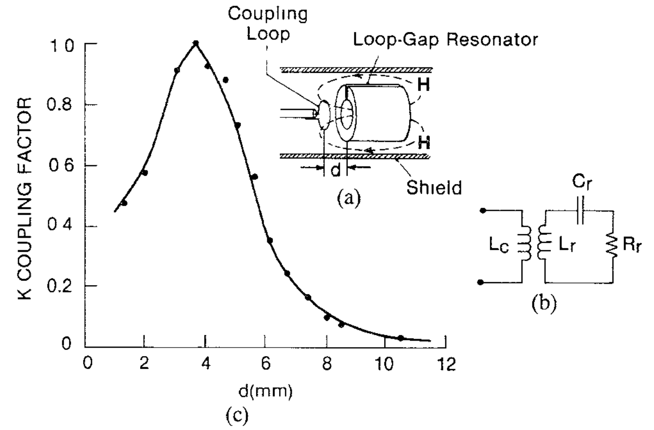

Fig. 2. Inductive loop coupling to a loop-gap resonator. (a) Coupling configuration. (b) Equivalent circuit. (c) Coupling coefficient versus loop distance.

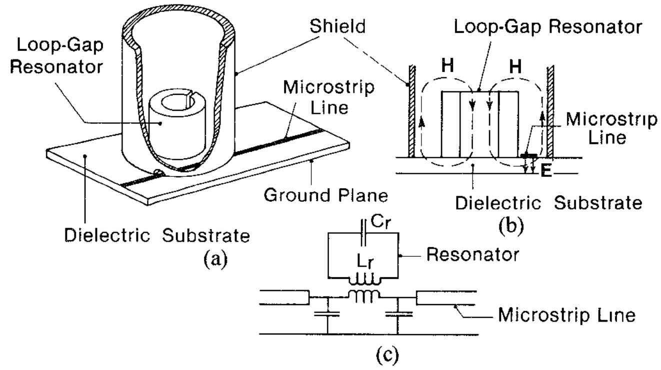

Fig. 3. Coupling of a loop-gap resonator to a microstrip line. (a) Schematic view of coupling. (b) Electromagnetic field configuration. (c) Equivalent circuit.

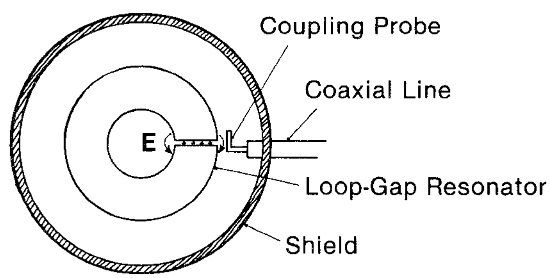

Fig. 4. Capacitive coupling to a loop-gap resonator by a monopole probe.

coupling coefficient of unity was obtained by this method for resonator No. 2 in Table I when a probe of 3-mm length was placed 1 mm from the gap.

Experiments on direct coupling of two loop-gap resonators placed uniaxially in a common shield show that the system resonates with two distinct resonant frequencies. Fig. 5 shows the two field configurations. The frequency response was measured by the use of a swept-frequency measurement setup which includes a sweep generator and a frequency counter coupled to the free end of one of the resonators through a coupling loop. Fig. 6 shows tuning curves for two resonant frequencies versus distance d between the resonators. Specifications of resonators used in

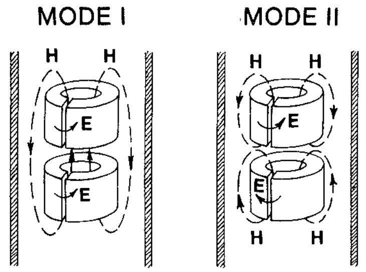

Fig. 5. Field configurations of the two modes for two coupled loop-gap resonators in a common shield.

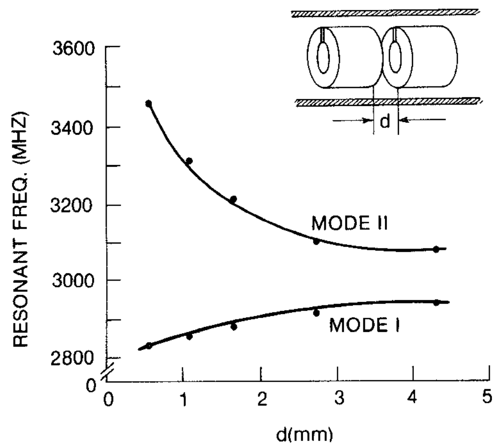

Fig. 6. Variation of frequencies of the two modes for two coupled loop-gap resonators, versus separation.

this test match those of resonator No. 2 in Table I. Identification of modes (correlation between modes in Figs. 5 and 6) was performed by partial insertion of a metallic plate in the transverse direction into the area between the two resonators. The effect of this perturbation is greater on Mode I (the even mode) than on Mode II (the odd mode). Direct coupling of uniaxial loop-gap resonators is the basis for multisection filters (Section V). It also can be used as a means of tuning the resonant frequencies.

III. Tunability

The resonant frequency of a loop-gap resonator can be tuned by mechanical or electronic means. An effective means of capacitive tuning is insertion of a dielectric slab in the gap, taking advantage of the gap's high electric-field intensity. Fig. 7 shows this tuning method and measured variations of the resonant frequency of resonator No. 2 in Table I versus the penetration of three dielectric slabs of different thicknesses and dielectric constants. The figure shows a tuning range of close to one octave by a slab with a dielectric constant of 10. Obviously, to maintain the quality factor over the tuning range, a very low-loss dielectric should be used. A different means of capacitive tuning that involves a conductive plate at the proximity of the gap is mentioned in [2].

Inductive tuning can be carried out by movement of a shorted loop at one end of the resonator [2]. The induced current on the loop gives a magnetic field opposed to the field in the resonator. This lowers the inductance and

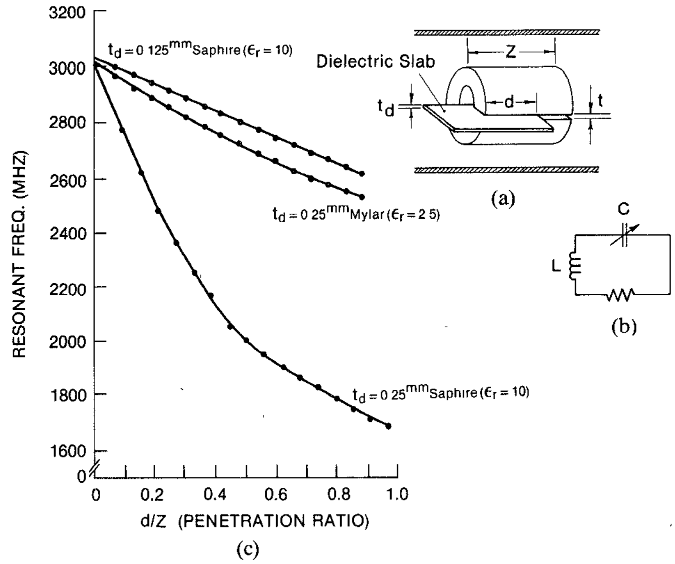

Fig. 7. Capacitive tuning of a loop-gap resonator. (a) Tuning configuration. (b) Equivalent circuit. (c) Tuning curves for three dielectric slabs.

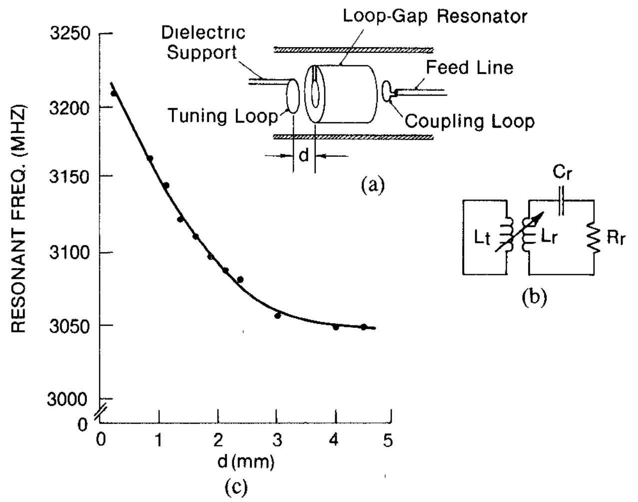

Fig. 8. Tuning of a loop-gap resonator by an inductive loop. (a) Tuning configuration. (b) Equivalent circuit. (c) Tuning curve.

increases the resonant frequency. Fig. 8 shows this tuning method with its equivalent circuit and a measured tuning chart for resonator No. 2 in Table I. The loop is 3 mm in radius and is made of a wire with a cross-sectional diameter of 1 mm. A wide tuning range is not possible with this method without degradation of resonance characteristics. Tuning by coupling to another loop-gap resonator was discussed in Section II.

A varactor can be used as a means for electronic tuning of a loop-gap resonator. Fig. 9 shows the coupling of a varactor to the gap by means of two coupling capacitances $C1$ and $C2$. The dc biasing circuit is isolated from the microwave fields by a quarter-wavelength coaxial cable terminated with a chip capacitor. The Q of the varactor is low at microwave frequencies, and therefore stronger coupling of the varactor to the gap for more tuning range would result in deterioration of the overall Q. For the 1-GHz resonator, No. 3 in Table I, and a GaAs varactor, a

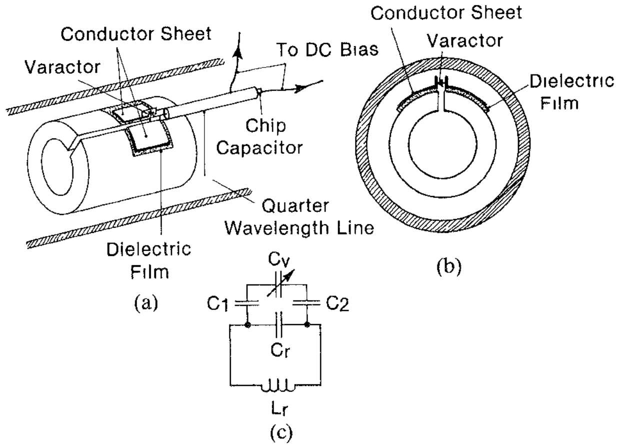

Fig. 9. Varactor tuning of a loop-gap resonator. (a) General view. (b) Cross-sectional view. (c) Equivalent circuit.

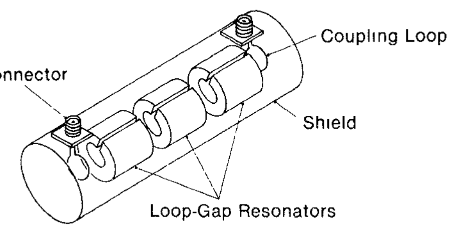

Fig. 10. Configuration of a three-element bandpass direct-coupled loop-gap filter.

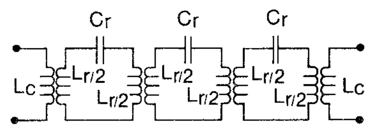

Fig. 11. Equivalent circuit of a three-element bandpass loop-gap filter.

tuning range of 30 MHz was obtained with an unloaded Q of 500. The size of the two conductor sheets was 10 mm by 10 mm, and the thickness of the dielectric film was 0.2 mm.

IV. Calculation of the Resonant Frequency and Q

A rough estimation of the resonant frequency of a loop-gap resonator can be carried out by considering its equivalent circuit as an LC-series circuit with the gap as a parallel-plate capacitor and the loop as a solenoid [3]. Such a method ignores the magnetic fringe fields at the two ends of the resonator and the gap's fringe electric fields, plus the effect of the shield. In the expression derived in [2], the shield effect is considered. In [3], an experimental correction factor is added for the fringe electric fields. In all of the calculation methods mentioned, the approximation of infinite length was used. The following expression has been developed [9] for the resonant frequency in which a factor for the effect of limited length of the resonator is added

$$f_0=\frac{c}{2\pi r_0}\sqrt{\frac{t}{\pi W}}\sqrt{1+\frac{r_0^2}{R^2-(r_0+W)^2}}\sqrt{\frac{1+\frac{\Delta Z}{Z}}{1+\frac{\Delta W}{W}}}. \tag{1}$$

In reference to Fig. 1 for geometrical parameters, r_0 is the inner loop radius, Z is the length of the resonator, W is the gap width, t is the gap distance, and R is the shield radius. The velocity of light is c. The parameter ΔZ is the equivalent length extension due to the magnetic fringe fields at two ends of the resonator. These fringe fields, shown in Fig. 1, are curved H-field lines that connect the H fields in the central region to the H fields at the annular region between the resonator and the shield. Similarly, ΔW is the equivalent length extension of the gap width due to the gap electric fringe fields.

Curve-fitting methods were used for 30 resonators, which vary in resonant frequency between 1–10 GHz, to find the following expressions for ΔZ and ΔW:

$$\Delta Z \simeq 0.18\,R \qquad \Delta W \simeq 3.0t. \tag{2}$$

Calculated values of resonant frequency, obtained by (1) and (2), are listed in Table I for comparison with measured results. The accuracy becomes poorer when the gap distance is large ($t/r_0 > 0.2$). Rigorous expressions are given in [9] for ΔZ and ΔW, which give better accuracy.

The following expression was found for the Q [9] by considering ohmic losses on walls of the resonator and the shield:

$$Q_0=\frac{r_0}{\delta}\,\frac{(1+p)\left(1+\frac{\Delta Z}{Z}\right)}{1+\left(1+\frac{W}{r_0}+\frac{R}{r_0}+\frac{R^2}{3.8Zr_0}\right)p^2} \tag{3}$$

where δ is the skin depth and p is given by

$$p=\frac{r_0^2}{R^2-(r_0+W)^2}. \tag{4}$$

Equation (3) reduces to the expression given in [2] when the infinite length approximation is used. Calculated values of Q using (3), and also taking account of ohmic capacitor losses [3, eq. (6)], are shown in Table I for comparison with measured results. Calculated values are slightly higher than measured values, possibly due to neglecting the higher order modes at the two ends of the resonator.

V. Filter Structures with Loop-Gap Resonators

Because of their coupling and tuning flexibility, loop-gap resonators can be used as resonant elements in compact filter structures. A bandpass filter can be designed by placing a number of resonators uniaxially in a common shield. Fig. 10 shows this structure. In Section II, coupling between two adjacent elements in such a structure was discussed. Because of the lumped-mode characteristics of a loop-gap resonator, existing theories for lumped-element filters can be used for design. An equivalent circuit for the filter of Fig. 10 is shown in Fig. 11. Detailed design procedures for such a lumped-element filter are given by Cohn in [10]. Fig. 12 shows a typical measured frequency response with figures of merit for a two-element bandpass direct-coupled loop-gap filter. This filter is 20 mm in

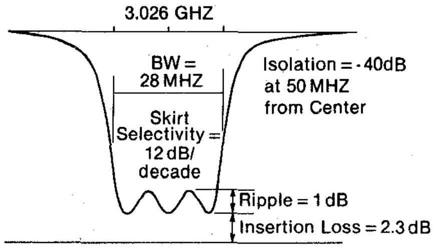

Fig. 12. Typical response of a two-element bandpass loop-gap filter.

cross-sectional diameter and 80 mm in length. The resonators are the same size as resonator No. 2 in Table I. The length of each resonator should be designed smaller than the passband wavelength in order to prevent spurious responses caused by TEM (coaxial) modes along the filter length.

VI. Oscillator Applications

The advantages that make a loop-gap resonator suitable for stabilization of microwave oscillators are its compactness, tunability, single-mode resonant operation, and freedom from spurious modes. With proper design of the active circuit, wide-range tunability of the resonator can be utilized to give oscillators comparable in tuning range to YIG-tuned oscillators. The coupling of the resonator to a microstrip line (Fig. 3) makes a suitable configuration for MIC oscillator design. Since the coupling configuration of Fig. 3 is similar to coupling of a dielectric resonator to a microstrip line, design methods for dielectric resonators [11], [12] can be used in the design of oscillators using loop-gap resonators. The procedure in [11] was followed by the authors in design of an oscillator that is tunable over the frequency range of 1.3–1.5 GHz using resonator No. 1 in Table I. An external Q of 600 was obtained.

VII. Discussion and Conclusion

It has been demonstrated that the class of resonators described here can be a design alternative between lumped and distributed resonator technologies for filter and oscillator applications, especially in the frequency band above UHF to 4 GHz where lumped-element resonant circuits become increasingly lossy and cavity resonators are cumbersome in practice. Dielectric resonators and SAW resonators have very limited tunability. The former become large below 1 GHz, and the latter are not practical above this frequency. The loop-gap resonator has been tested at X-band (see resonator No. 4 in Table I). Magnetic resonance spectroscopy is now being routinely performed at X-band using loop-gap resonators; so it is likely that they are useful in that frequency range for other applications. It is concluded that loop-gap resonators can be used in many applications where a medium-range Q is sufficient and a small size is important.

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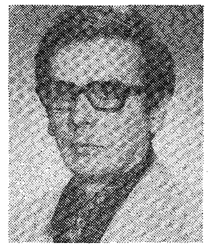

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